Worksheet 22 Solutions.

1) a) A parameterization of a curve encodes more information than just the set of (x,y) points that lie on the curve: for example orientation, speed, # of times the curve is traversed, etc.

b) We want $x=f(t)$, $y=g(t)$ so that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

We have $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$ and we know that $\cos^2(t)+\sin^2(t)=1$,

so maybe $\frac{x}{a}=\cos(t)$ and $\frac{y}{b}=\sin(t)$, i.e. $\boxed{\begin{array}{ll} x=a\cos(t) & \text{for } t \text{ in } [0,2\pi] \\ y=b\sin(t) & \end{array}}$

Graphing these parametric expressions shows that the does cover the whole ellipse.

c) No. Although both parametrize the circle $x^2+y^2=25$ (plug in & check), the first set traverses this circle once counterclockwise on $[0,2\pi]$, while the second set traverses the circle ten times counterclockwise on $[0,20\pi]$.

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2) a) We have $x=2\cos t$, $y=2\sin t$. Again, let's use $\cos^2(t)+\sin^2(t)=1$ to relate these:

$x^2+y^2=4\cos^2(t)+4\sin^2(t)=4(\cos^2(t)+\sin^2(t))=4(1)=4$,

so $\boxed{x^2+y^2=4}$ is our solution, where $-2\le x\le 2$, $-2\le y\le 2$.

b) $x=3t-5$, so $\frac{x+5}{3}=t$. So $\boxed{y=\frac{2}{3}(x+5)+3}$

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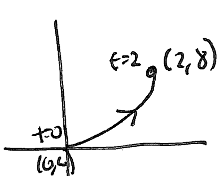
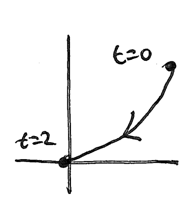
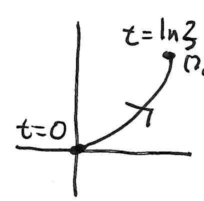
3) Some possible answers



$x=t$
$y=t^3$
t in $[0,2]$



$x=2-t$
$y=(2-t)^3$
$0\le t\le 2$



$x=e^t-1$
$y=(e^t-1)^3$
$0\le t\le \ln 3$

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4) One idea: choose $\boxed{x=t+3}$, which satisfies $x(0)=3$. Then $y=x^2$ forces $\boxed{y=(t+3)^2}$ which satisfies $y(0)=9$.

Another option: $\boxed{\begin{array}{l} x=\cos(t)+2, \\ y=(\cos(t)+2)^2 \end{array}}$ which also satisfies $x(0)=3$. Then because $y=x^2$, we get $=\cos^2(t)+4\cos(t)+4$, which satisfies $y(0)=9$.

5] Relate sec(t), tan(t) using Pyth. Thm:

$$1 + \tan^2(t) = \sec^2(t).$$

Since $x = \sec(t)$, $y = \tan(t)$ for $0 \le t \le \pi/2$,

this gives $1 + y^2 = x^2$ for $\sec(0) \le x \le \sec(\pi/2)$

i.e. $y^2 = x^2 - 1$ for $1 \le x < \infty$

Since $y = \tan(t) \ge 0$ for $0 \le t \le \pi/2$,

$\boxed{y = \sqrt{x^2 - 1} \text{ for } 1 \le x < \infty}$ is the Cartesian equation of this curve

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6] Because the particle travels in a straight line, ~~x(t) and y(t) will~~ the easiest parameterization is to take $x(t), y(t)$ to be linear equations:

At $t=0$, $x=2$; at $t=5$, $x=1$: So $x(t) - 2 = \frac{1-2}{5-0}(t-0)$

$$\boxed{x = -\frac{1}{5}t + 2}$$

At $t=0$, $y=3$. At $t=5$, $y=-1$. So $y(t) - 3 = \frac{-1-3}{5-0}(t-0)$

$$\boxed{y = -\frac{4}{5}t + 3}$$